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DYNAMIC MINIMUM BICHROMATIC SEPARATING CIRCLE

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Abstract— Consider two point sets in the plane, a red set and a blue set. We show how to find the smallest circle that encloses all red points and as few blue points as possible in its interior, using linear programming with violations.

Keywords— Voronoi diagram , linear separability.

I. INTRODUCTION

We study a circular separability problem in R^2 , which we refer to as the minimum separating circle problem. Consider two point sets R and B, and let S denote the set of circles such that each circle in S encloses all points in R and has the minimum possible number of points of B within its interior. The problem asks to find the smallest circle in S. In other words, we want to find the smallest circle that encloses all red points and contains as few blue points as possible in its interior. See Fig. I for an illustration. We denote the minimum separating circle by $C_B(R)$. The problem arises in classification of point sets that are not purely separable.

Two point sets are circularly separable if there exists a circle that contains all points of one set and no point of the other set in its interior. Given two circularly separable point sets, Fisk studied the optimization version of the circular separability problem, i.e. computing a separating circle with minimum radius, and gave a quadratic time and space algorithm using both the nearest point and farthest point Voronoi diagrams [3]. The result was improved to optimal linear time and space in [5]. The improvement was achieved by reducing the planar circu- lar separability problem to a linear separability problem in R^3 . The authors observed that after lifting all points in the plane to a paraboloid in R^3, each plane that linearly separates the lifted point sets corresponds to a circle that separates the original point sets on the xy- plane, and vice versa. In the same paper, they also gave an optimal $O(N \log N)$ time algorithm for finding the largest separating circle, where N is the total number of points.

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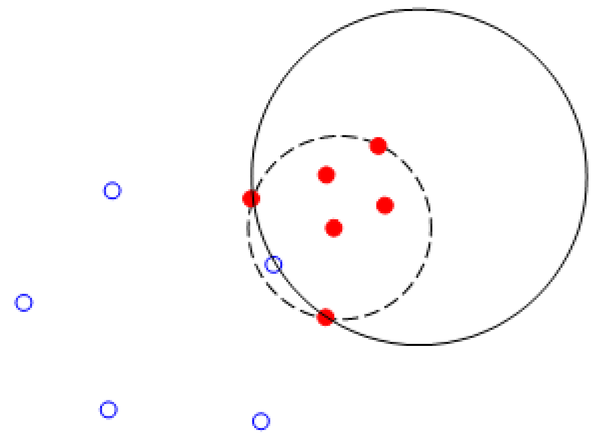

Figure 1. The minimum enclosing circle of red points (dashed) and the minimum separating circle (solid).

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For linear separability, in which point sets are sepa- rated by hyper-planes, it is well known that the problem reduces to linear programming, which in turn can be solved in linear time in any fixed dimension using Megiddo's solution [4]. Aronov et. al. [1] considered the linear separability problem for point sets that may be linearly inseparable, in arbitrary dimensions. When the points are linearly partitioned into two parts, each part may contain some misclassified points. The authors defined four different metrics for measuring the error of classification based on the amount of work needed to move the misclassified points and gave solutions for finding the best separator which minimizes the error un- der each metric. For one of the metrics, which measures the number of misclassified points, the authors gave an $O(N^d)$ time algorithm where $d \geq 2$ is the dimension of the space. Chan [2] gave an alternative solution for this problem in two and three dimensions by solving linear programming with violations. Chan's approach is based on concave-chain decomposition of the ($\leq k$)-level. His algorithm takes $O((N + k^2) \log N)$ expected time and $O(N + k^{11/4} N^{1/4} \log^{O(1)} N)$ expected time in R^2 and R^3 , respectively, where k is the number of misclassified points.

In [6], we presented two deterministic solutions for the minimum separating circle problem, which take $O(nm \log m + n \log n)$ time and $O((n + m) \log n) + O^*(m^{1.5})$

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time, respectively, where n is the number red points, m is the number of blue points, and the notation $O^*(\ )$ ignores a polylogarithmic factor. The first approach is based on a sweep of the farthest point Voronoi diagram of the red points. The second one is based on circular range queries where query points lie on the edges of the farthest Voronoi diagram of the red points. In this paper, we present an alterna- tive solution using O'Rourke's algorithm and Chan's solution for linear programming with violations, which takes $O((N + k^{11/4} N^{1/4} \log^{O(1)} N)N)$ expected time.

The second solution, which takes sub-quadratic time, generally outperforms the other two. The new solution is output sensitive. It takes near quadratic time and outperforms the first solution when k is small, e.g. $K = O(1)$.

In *Section* 2, we briefly discuss O'Rourke's approach, which solves the minimum separating circle problem for separable point sets. In *Section* 3, we discuss a solution for the general case by applying Chan's approach for linear programming with violations.

II. COMPUTING A SEPARATING CIRCLE

O'Rourke et. al. [5] gave an optimal linear time so- lution for determining circular separability. The authors observed that a plane cuts a paraboloid $P : z = x^2 + y^2$ in an ellipse and the projection of this ellipse on the xy- plane is a circle (See Fig. II). Hence, the separating circle problem in R^2 reduces to a linear separability problem in R^3 by geometrically lifting the points to a paraboloid.

Let $R = \{(x_i, y_i) : i \in I_1\}$ and $B = \{(x_i, y_i) : i \in I_2\}$ be two circularly separable point sets, where I_1 and I_2 are index sets for R and B, respectively. First, these two sets are geometrically lifted to the paraboloid $P : z = x^2 + y^2$ by the mapping $(x, y) \to (x^2, y^2, x^2 + y^2)$. Let $R^{\prime}$ and $B^{\prime}$ be the lifted point sets for R and B, respectively.

The equation of the projection of the intersection between the plane $ax + by + z = c$ and the paraboloid

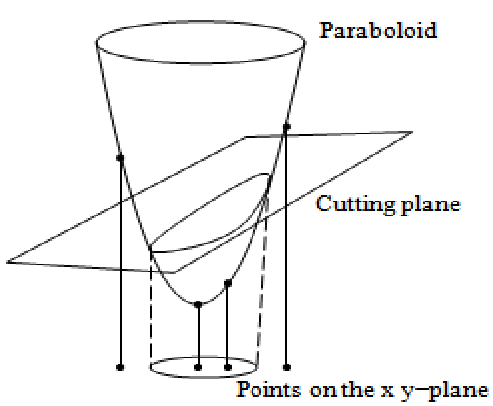

Figure 2. Intersection between a plane and a paraboloid.

$P : z = x^2 + y^2$ on the xy-plane is:

$$ax + by + (x^2 + y^2) = c,$$

or

$$(x + \frac{1}{2}a)^2 + (y + \frac{1}{2}b)^2 = c + \frac{1}{4}(a^2 + b^2).$$

Similarly, it can be shown that the mapping of all points on the circle $(x - A)^2 + (y - B)^2 = R^2$ on the xy-plane to the paraboloid $P : z = x^2 + y^2$ lies on a plane:

$ax + by + z = c,$

where $a = -2A$, $b = -2B$, and $c = R^2 - \frac{1}{4}(a^2 + b^2)$. Since there is a one-to-one correspondence between plane in R^3 intersecting the paraboloid and a circle on the xy-plane the authors claimed the following property:

Given a circle $(x - A)^2 + (y - B)^2 = R^2$ and a point (x_i, y_i), the circle encloses the point if and only if

where $a = -2A$, $b = -2B$, and $c = R^2 - \frac{1}{4}(a^2 + b^2)$. Hence, the optimization version of the separating circle problem can be formulated as a convex quadratic pro- gramming problem with linear constraints (the decision version can be formulated as problem):

$$\min_{a,b,c} \frac{1}{4}(a^2 + b^2) + c$$

subject to

$$ax_i + by_i + (x_i^2 + y_i^2) \leq c \ (i \in I_1),$$ and

$$ax_i + by_i + (x_i^2 + y_i^2) \geq c \ (i \in I_2).$$

Both the decision version and the optimization version can be in turn solved by Megiddo's solution in linear time [4].

III. SEPARABILITY USING LINEAR PROGRAMMING WITH VIOLATIONS

The linear separability with at most k violations in d dimensions is defined as follows: Given a set H of linear constraints or closed half-spaces in R^d, a linear objective function f and a positive integer number k, minimize f over the union of points which lie outside at most k half-spaces in H. Let $I_k(H)$ be the domain of this problem, i.e. the union of points which lie outside at most k half-spaces in H. In [2], Chan presented an efficient algorithm for this problem for $d = 2$ and $d = 3$. The outline of Chan's algorithm is surprisingly simple. The author first computes the $(\leq k)$-level of the lower half-spaces of H, which is the collection of the 0-level, 1-level, . . ., k-level of the lower half-spaces, and the $(\leq k)$-level of the upper half-spaces. Then, all $O(k^d)$ local minima of $I_0(H), I_1(H), \ldots, I_k(H)$ are computed by checking the intersection points of the two structures. In order to improve the running time, the author does not work directly with the $(\leq k)$-level, which has complexity $O(n^{\lfloor d/2 \rfloor} k^{\lceil d/2 \rceil})$, but instead works with its convex/concave a linear programming chain decomposition (resp.

convex/concave-surface de- composition in R^3), which contains $O(k)$ chains/surfaces of total size $O(n)$.

As discussed in the previous section, the general separating circle problem reduces to a convex quadratic programming problem with linear constraints in R^3 . Although, Chan's solution can be extended to solve two dimensional convex quadratic programming, its applica- tion to R^3 is unclear. To find the smallest separating circle as defined earlier, for points that cannot be cir- cularly separated, we first apply Chan's solution on the decision version of the separating circle problem, then apply O'Rourke's algorithm [5] to find the exact optimal solution. We define the decision version as follows:

Given two finite planar sets R and B and a constant k, decide if there exists a circle that encloses R and contains no more than k points from B in its interior. We can formulate this problem as a linear programming in R^3 :

Note that the minimum separating circle contains no more than k blue points if and only if the corresponding linear programming problem has a solution with at most k violations.

Applying Chan's algorithm, we can answer the deci- sion problem in $O(N + k^{11/4} N^{1/4} \log^{O(1)} N)$ expected time. Chan's algorithm enumerates all local minima of $I_k(H)$. Each local minimum defines a plane which corresponds to a separating circle which misclassifies at most k points. Hence, for each local minimum generated by Chan's algorithm, we remove the points associated

with constraints violated, i.e. points misclassified. The optimal separating circle for the remaining points can be found in linear time using O'Rourke's solution [5], since the remaining points are circularly separable. To find the minimum k value for the separating circle we can guess

k from a doubling sequence, e.g. set $k = 1, 2, 4, \ldots$. As a result, we can compute the minimum separating circle in $O((N + k^{11/4} N^{1/4} \log^{O(1)} N)N)$ expected time. The additional N factor is due to finding the minimum k value from the doubling sequence and the running time of O'Rourke's solution [5] at each local minimum of $I_k(H)$.

Notice that since all red points have to be enclosed by the circle, not all constraints can be violated. After the reduction, in the constraint set H which consists of a set of upper half-spaces and lower half-spaces, all red points contribute to upper half-spaces of H and all blue points contribute to lower half-spaces H. Thus, we simply compute the 0-level of upper half-spaces instead of the $(\leq k)$-level.

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