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SOLVING FUZZY TRAPEZOIDAL GAME THEORY PROBLEM USING DOMINANCE PROPERTY IN DIFFERENT RANKING FUNCTION METHODS

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Abstract— A variety of methods are proposed for solving Game theory problems but all the methods in literature, demonstrate the parameters by normal fuzzy numbers. In most of the work it is quite impossible tolimit the membership function to the normal form. So in this paper we deal with games of fuzzy payoffs problems while there is uncertainty in data.We use the trapezoidal fuzzy numbers which make the data fuzziness and utilize the new proposed ranking function algorithm by for using trapezoidal fuzzy numbers the decision maker to get the best gains.

Keywords:
Fuzzy numbers.Game Theory problem, Fuzzy Trapezoidal numbers,RankingTechnique,Players,Payoff Matrix.

I. INTRODUCTION

The theory of games is a mathematical theory that deals with general features of competitive situations. It is usually used when two or more individuals ororganizations with conflicting objectives try to make decisions, it's based on the minimax-principle The set of objectivefunctions in the game may have uncertain values where the way to deal with uncertainty is touse the concept of fuzzy games . A ranking function is used which helps us not just to find the solution but also to find the best gain for the fully fuzzy game problem.

Many authors studied fuzzy games some of them with ranking functions to solve the game problem. Aristuidou.M,Sarangi.Sat(2005)representedanon-cooperative model of a normal form game using tools from fuzzy settheory. Gao.Jat(2007) represented a strategic game with fuzzy payoffs. Medinechiene. M, Zavadskas. E. K, Turskis. Z at (2011) described a model of dwelling selection using fuzzy games theory on buildings. Jawad.M.Aat(2012) represented fuzzysets and fuzzy processes with game theory to address the uncertainty in data for mobile phone companies in Iraq . Kumar. R. S, Kumaraghura. S at (2015) represented a solution of fuzzy game problem with triangular fuzzy numbers using a ranking function tocompare the fuzzy numbers Selvakumari. K, Lavanya. S at (2015) considered a two zero sum game with imprecise (triangularortrapezoidal) fuzzy numbers using ranking function for an approach to solve the problem .Kumar.R.S, Gnanaprakash.K at(2016) representeda(3×3) two zero sum game with octagonal fuzzy payoffs using ranking function to solvethefuzzy game.

The objective of this paper is to propose a new algorithm depending on a ranking function to solve the fuzzy game problem using trapezoidal fuzzy numbers and trying to get a desirable gain.This paper contains six sections : insection one and two abstract and introduction of some theory concepts, section three defines the fuzzy definition and , section four represents theranking function and some properties and anew proposed algorithm, finally in section five anumericalexampleis represented dependingon thenewproposed algorithm last section conclusion for future work are described in Section.

II. PRELIMINARIES :

Fuzzy set theory was introduced by Zadeh [2]. It is widely used in almost all areas to mathematically describe uncertainty and vagueness and to give formalised tools for dealing with the imprecision intrinsic to many problems. Here, we present some basic definitions of fuzzy set, fuzzy number, trapezoidal fuzzy number, generalised trapezoidal fuzzy number and the arithmetic operations of generalised trapezoidal fuzzy numbers.

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Definition 1: A fuzzy set $\tilde{A}$, defined on the universal set X

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0, 1]$ is the membership function such that

$\mu_{\tilde{A}}(x) = 0$ if x does not belong to A, $\mu_{\tilde{A}}(x) = 1$ if x strictly belongs to A.

Definition 2: A fuzzy set $\tilde{A}$, defined on the universal set of real numbers ^{R}, is said to be fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x) : {}^R \to [0, 1]$ is continuous,

(ii) $\mu_{\tilde{A}}(x) = 0$ *for all* $x \in (-\infty, a] \cup [d, \infty)$,

(iii) $\mu_{\tilde{A}}(x)$ *is strictly increasing on [a,b] andstrictly decreasing on [c,d]*, (iv) $\mu_{\tilde{A}}(x) = w$ *for all* $x \in [b, c]$, *where* $0 < w \leq 1$.

Definition 3 : A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy if its membership function $\mu\tilde{A}(x) = \{$ b - $\frac{1}{x-a}$, if a $\leq x \leq b$

$\frac{x-1}{c-d}$, if c $\leq\leq x \leq\leq d$

0, elsewhere }

Definition 4 :: A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be ageneralised trapezoidal fuzzy number if its membership function is stated as shown below:

Trapezoid (w ; a,b,c,d) = { 0 , if w $\leq$ a

$\frac{w-a}{b-a}$, if a $\leq$ w $\leq$ b

1 , if b $\leq$ w $\leq$ c

$\frac{d-w}{d-c}$, if c $\leq$ w $\leq$ d }

III. GENERALIZED TRAPEZOIDAL FUZZY NUMBER :

Ranking function : 1

A new concept for ranking of generalized trapezoidal fuzzy number is presented using trapezoid as reference point. This Ranking methods map fuzzy number directly in to the real line. To Find

R (A) = [(a1+b1+c1+d1)/4]

Ranking function :2

Ranking of fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. In fuzzy multi-attribute decision making, the ratings and decision-maker's risk performance are calculated based on fuzzy numbers. When the information is disclosed as a fuzzy number, it is necessary to calculate and compare fuzzy data before a decision is made. Therefore, ranking fuzzynumbers is a usefulapproach fortacklingdecision making problem. Many methods have been introduced to deal with the ranking of fuzzy numbers. However, the methods have never been simplified and tested in real-life applications. In the recent decades, many researchers have investigated various ranking functions. So, we can see that the task in ranking of fuzzy numbers is very important in the research topic of fuzzy decision making. Fuzzy number must be ranked before an action is taken by a decision maker. By ranking of fuzzy numbers, we can rank alternatives and find the best one from them. In this paper, we define a ranking function, $\Re(\tilde{A})$ for generalised trapezoidal fuzzy number, $\tilde{A} = (a, b, c, d; w)$ as follows:

$$\Re(\tilde{A}) = w \cdot \frac{2a_1 + b_1 + c_1 + 2d_1}{6} \quad (1)$$

Since ranking function is an approach for comparing two generalised trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ is defined as $\Re : F(\mathbb{R}) \to \mathbb{R}$ where $F(^R)$ is a set of fuzzy numbers defined on the universal set of real numbers which map for each fuzzy number into the real line. Thus specific ranking of fuzzy numbers is an important procedure for decision making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory. Then

(i) $\tilde{A}_1 \tilde{\leq} \tilde{A}_2$, *if andonlyif* $\Re(\tilde{A}_1) = w \cdot ((2a_1 + b_1 + c_1 + 2d_1)/6) \leq \Re(\tilde{A}_2) = w \cdot ((2a_2 + b_2 + c_2 + 2d_2)/6)$, $w = \min\{w_1, w_2\}$,

(ii) $\tilde{A}1 \tilde{\geq} \tilde{A}2$, *ifandonly if* $\Re(A_1) = w \cdot ((2a_1 + b_1 + c_1 + 2d_1)/6) \geq \Re(A_2) = w \cdot ((2a_2 + b_2 + c_2 + 2d_2)/6)$, $w = \min\{w_1, w_2\}$,

(iii) $\tilde{A}1 \sim= \tilde{A}2$, *ifandonly if* $\Re(A_1) = w \cdot ((2a_1 + b_1 + c_1 + 2d_1)/6) = \Re(A_2) = w \cdot ((2a_2 + b_2 + c_2 + 2d_2)/6)$, $w = \min\{w_1, w_2\}$.where the symbols $\tilde{\leq}$, $\tilde{\geq}$ and $\sim=$ are chosen as fuzzyrelations.

IV. SOME BASIC GAME CONCEPT:

1)Matrix Game with Payoff:

In the traditional game problem, it is assumed that the decision makers have an idea of how payment should be made at the end of the table. In real-world problems, the payoffs may not be known precisely by the decision makers. Then some sorts of uncertainty arise about the payoffs. Therefore, the fuzzy set theory is applied to accommodate such types of matrix game problems. Here, we consider a matrix game $A = (a_{ij})_m \times n$ and assume that $S_1 = \{\alpha_i : i = 1, 2, \dots, m\}$ and $S_2 = \{\beta_j : j = 1, 2, \dots, n\}$ are the sets of pure strategies of the players I and II, respectively. Also let X and Y be the mixed strategies for the players I and II, respectively, where $X = \{x_i : i = 1, 2, \dots, m\}$ and $Y = \{y_j : j = 1, 2, \dots, n\}$. Then a simple two-person zero-sum matrix game can be expressed with the triplet $G = <X, Y, A>$.

Here, the existing methods, such as Concept of dominance, Graphical method, Algebraic method and Simplex method, are used to calculate the optimal strategy and the value of the matrix game when the payoffs are generalised trapezoidal fuzzy numbers. Assume that a matrix game with generalised trapezoidal fuzzy payoffs is given in Table 1.

First, we define the rank of each generalised trapezoidal fuzzy number by using the proposed ranking function. This supports us to reduce the payoff matrix with generalised trapezoidal fuzzy payoffs into an equivalent crisp game problem and we solve it by the existing methods. So, the corresponding crisp game problem is defined in Table 2.

2)Two person zero sum games and pay-off matrix :

In this section we give some basic definitions of the two person zero sum games and pay-off matrix. These concepts form the basic building blocks of game theory.

3) Two person zero sum games :

A game of two persons in which gains of one player are losses of other player is called a two person zero sum game i.e. in two person zero sum the algebraic sum of gains to both players after a play is bound to be zero. Games relating to pure strategies taken by players are considered here based on two assumptions :

Player A is in better position and is called maximization player (or row player) and player B is called minimizing player (or column player).

Total gain of one player is exactly equal to total loss of otherplayer.Ingeneral,ifplayer A takes m purestrategies and B takes n pure strategies, then the game is called two person zerosum game or $m \times n$ rectangular game.

4) Solution of 2×2 games with mixed strategies :

Consider the fuzzy game [1, 4, 6] of players A (strategies represented horizontally) and B (strategies represented vertically) whose pay-off is given by following matrix and for which there is no saddle point.

$$\begin{bmatrix} <a_{11}, b_{11}> & <a_{12}, b_{12}> \\ <a_{21}, b_{21}> & <a_{22}, b_{22}> \end{bmatrix}$$

where, pay-off $<a_{ij}, b_{ij}>$ are symmetric LR-type trapezoidal fuzzy numbers such that $b_{ij} = a_{ij} + \alpha$. If x_i and y_j be the probabilities by which A chooses i^{th} strategy and B chooses jth strategy.

5) Games with no saddle point :

The simplest case is a 2 × 2 Fuzzy game with no saddle point. Here, we consider a $m \times n$ fuzzy game. Now we discuss a particular method. In this method, the pay-off can be reduced to 2 × 2 games so that it can be solved by using the fuzzy game method. The method of reduction of the pay-off matrix by this process is called the dominance property of the rows and columns of the pay-off matrix.

6)Concept of dominance :

If one pure strategy of a player is better for him or as good as another, for all possible pure strategies of opponent then first is said to dominate the second. The dominated strategy can simply be discarded from pay-offmatrix.

Since ith as no value. When this done,optimal strategies for the reduced matrix are also optimal for the original matrix with zero probability for discarded strategies. When there is no saddle point in pay-off matrix, then size of the game can be reduced by dominance, before the problem is solved.

Definition 1 : If all elements of the i^{th} row of pay-off matrix of a $m \times n$ rectangular game are dominating over r^{th} row in the sense of maximization, r^{th} row is discarded and deletion of r^{th} row from matrix does not change the set of optimal strategies of maximizing player.

Definition 2 : If all elements of the j^{th}column of pay-off matrix of a $m \times n$rectangular game are dominating over j^{th}column in the sense of minimization, j^{th}column is discarded and deletion of j^{th}column from matrix does not change the set of optimal strategies of minimizing.

7) Fuzzy Game problem:

TheNumerical fuzzygame problemis whereall thepayoffsofthegamematrixarefuzzyquantities.Nowtheformulaof thefullyfuzzygame problemis as follows

Ranking function 1Table :R (A) = [(a1+b1+c1+d1)/4]

	Player B				
Player A	(1,2,3,4)	(7,3,4,3)	(2,2,3,4)	(3,2,5,4)	(5,3,2,3)
	(5,3,2,5)	(1,1,3,2)	(2,3,4,4)	(2,3,2,2)	(8,7,3,1)
	(3,5,2,1)	(7,6,5,4)	(2,2,1,5)	(5,67,3)	(4,7,5,3)
	(3,4,7,3)	(6,8,5,7)	(7,3,2,1)	(3,7,7,1)	(5,6,3,8)

By using the ranking procedure the trapezoidal fuzzy Game problem is reduced to the normal fuzzy Game problem.

2.5	4.25	2.75	3.5	3.25
3.75	1.75	3.25	2.25	4.75
2.75	5.5	2.5	5.25	4.75
4.25	6.5	3.25	4.5	5.5

8)Dominance property method :

First and second row is dominatedbythe fourth row .Omit the first and second row.The reduced game is

2.75	5.5	2.5	5.25	4.75
4.25	6.5	3.25	4.5	5.5

Second, fourth and fifth column is dominated by the first column . Omit the Second, fourth and fifth column.The reduced game is

2.75	2.5
4.25	3.25

First row is dominated by the second row . Omit the first row . The reduced game is

4.25	3.25

First column is dominated by the Second column . Omit the First column . The reduced game is

3.25

The value of the game for the example is v = 3.25

Ranking function 2

Table : R (A) = w.[(2a1+b1+c1+2d1)/6] ,w $\in$ (0,1)

Take w = 0.1

	Player B				
Player A	(1,2,3,4)	(7,3,4,3)	(2,2,3,4)	(3,2,5,4)	(5,3,2,3)
	(5,3,2,5)	(1,1,3,2)	(2,3,4,4)	(2,3,2,2)	(8,7,3,1)
	(3,5,2,1)	(7,6,5,4)	(2,2,1,5)	(4,7,5,3)	(5,67,3)
	(3,4,7,3)	(6,8,5,7)	(7,3,2,1)	(3,7,7,1)	(5,6,3,8)

By using the ranking procedure the trapezoidal fuzzy Game problem is reduced to the normal fuzzy Game problem.

0.25	0.45	0.28	0.35	0.32
0.38	0.16	0.32	0.18	0.5
0.28	0.55	0.28	0.5	0.55
0.35	0.65	0.35	0.47	0.58

First row is dominatedby the fourth row .Omit the first row.The reduced game is

0.38	0.16	0.32	0.18	0.5
0.28	0.55	0.28	0.5	0.55
0.35	0.65	0.35	0.47	0.58

Second row is dominated by the Third row . Omit the Second row.The reduced game is

0.38	0.16	0.32	0.18	0.5
0.35	0.65	0.35	0.47	0.58

Fifth column is dominated by the Firstcolumn. Omit the Fifthcolumn . The reduced game is

0.38	0.16	0.32	0.18
0.35	0.65	0.35	0.47

First column is dominated by the Third column. Omit the First column . The reduced game is

0.16	0.32	0.18
0.65	0.35	0.47

First row is dominatedby the Second row .Omit the first row.The reduced game is

0.65	0.35	0.47

First and Third column is dominated by the Second column. Omit the First and third column . The reduced game is

0.35

The value of the game is v = 0.35

V. CONCLUSION

Then the new proposed algorithm will be helpful for the decision maker when he deals with a fuzzy game problem to get the best gain. We considered the solution of rectangular fuzzy games using trapezoidal fuzzy numbers. Here pay-off is considered a simprecise number I instead of crispnumbers which takes care of the uncertainty and vagueness inherent in such problems. Trapezoidal fuzzy numbers are used becauseof their simplicity and computational efficiency. We discuss a solution of fuzzy games with pure strategies by minimax-maximin principle and also algebraic method to solve 2 × 2 fuzzy games without saddle point by using mixed strategies.The concept of dominance method is also illustrated. Trapezoidal fuzzy numbers generates optimal solutions which are feasible in nature and also takes care of the impreciseness aspect.

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