CHAPTER 11
An Introduction to set theory and its Applications in Engineering

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ABSTRACT

Set theory is the branch of mathematical logic that studies sets, which can be informally described as collections of objects. Although objects of any kind can be collected into a set, set theory — as a branch of mathematics — is mostly concerned with those that are relevant to mathematics as a whole. The modern study of set theory was initiated by the German mathematicians Richard Dedekind and Georg Cantor in the 1870s. In particular, Georg Cantor is commonly considered the founder of set theory. The non-formalized systems investigated during this early stage go under the name of naive set theory. After the discovery of paradoxes within naive set theory (such as Russell's paradox, Cantor's paradox and the Burali-Forti paradox), various axiomatic systems were proposed in the early twentieth century, of which Zermelo–Fraenkel set theory (with or without the axiom of choice) is still the best-known and most studied. Set theory is commonly employed as a foundational system for the whole of mathematics, particularly in the form of Zermelo–Fraenkel set theory with the axiom of choice. Besides its foundational role, set theory also provides the framework to develop a mathematical theory of infinity, and has various applications in computer science (such as in the theory of relational algebra), philosophy, formal semantics, and evolutionary dynamics[1]. Its foundational appeal, together with its paradoxes, its implications for the concept of infinity and its multiple applications, have made set theory an area of major interest for logicians and philosophers of mathematics. Contemporary research into set theory covers a vast array of topics, ranging from the structure of the real number line to the study of the consistency of large cardinals.

Keywords: *Set theory , Zermelo–Fraenkel set theory, Burali-Forti paradox etc*

INTRODUCTION

ISBN: 978-93-5980-742-3, www.aetsjournal.com — Handbook of research in Basic and Applied Science

A set is pure if all of its members are sets, all members of its members are sets, and so on. For example, the set containing only the empty set is a nonempty pure set. In modern set theory, it is common to restrict attention to the von Neumann universe of pure sets, and many systems of axiomatic set theory are designed to axiomatize the pure sets only. There are many technical advantages to this restriction, and little generality is lost, because essentially all mathematical concepts can be modeled by pure sets. Sets in the von Neumann universe are organized into a cumulative hierarchy, based on how deeply their members, members of members, etc. are nested. Each set in this hierarchy is assigned (by transfinite recursion) an ordinal number

Georg Cantor (1845-1918), a German mathematician, initiated the concept 'Theory of sets' or 'Set Theory'. While working on "*Problems on Trigonometric Series*", he encountered sets, that have become one of the most fundamental concepts in mathematics. Without understanding sets, it will be difficult to explain the other concepts such as relations, functions, sequences, probability, geometry, etc.

DEFINITION OF SETS

As we have already learned in the introduction, set is a well-defined collection of objects or people. Sets can be related to many real-life examples, such as the number of rivers in India, number of colours in a rainbow, etc.

EXAMPLE

To understand sets, consider a practical scenario. While going to school from home, Nivy decided to note down the names of restaurants which come in between. The list of the restaurants, in the order they came, was:The above-mentioned list is a collection of objects. Also, it is well-defined. By well-defined, it is meant that anyone should be able to tell whether the object belongs to the particular collection or not. E. g. a stationary shop can't come in the category of the restaurants. If the collection of objects is well-defined, it is known as a set.The objects in a set are referred to as elements of the set. A set can have finite or infinite elements. While coming back from the school, Nivy wanted to confirm the list what she had made earlier. This time again, she wrote the list in the order in which restaurants came. The new list was: Now, this is a different list. But is a different set? The answer is no. The order of elements has no significance in sets so it is still the same set.

REPRESENTATION OF SETS

Sets can be represented in two ways:

1. Roster Form or Tabular form
2. Set Builder Form

ROSTER FORM

In roster form, all the elements of the set are listed, separated by commas and enclosed between curly braces { }.

Example: If set represents all the leap years between the year 1995 and 2015, then it would be described using Roster form as:

A ={1996,2000,2004,2008,2012}

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Now, the elements inside the braces are written in ascending order. This could be descending order or any random order. As discussed before, the order doesn't matter for a set represented in the Roster Form.

Also, multiplicity is ignored while representing the sets. E.g. If **L** represents a set that contains all the letters in the word ADDRESS, the proper Roster form representation would be

L ={A,D,R,E,S }= {S,E,D,A,R}

L≠ {A,D,D,R,E,S,S}

SET BUILDER FORM

In set builder form, all the elements have a common property. This property is not applicable to the objects that do not belong to the set.

Example: If set **S** has all the elements which are even prime numbers, it is represented as:

S={ x: x is an even prime number}

where 'x' is a symbolic representation that is used to describe the element.

':' means 'such that'

'{}' means 'the set of all'

So, S = { x:x is an even prime number } is read as 'the set of all x such that x is an even prime number'. The roster form for this set S would be S = 2. This set contains only one element. Such sets are called singleton/unit sets.

Another Example:

F = {p: p is a set of two-digit perfect square numbers}

How?

F = {16, 25, 36, 49, 64, 81}

We can see, in the above example, 16 is a square of 4, 25 is square of 5, 36 is square of 6, 49 is square of 7, 64 is square of 8 and 81 is a square of 9}.

Even though, 4, 9, 121, etc., are also perfect squares, but they are not elements of the set F, because the it is limited to only two-digit perfect square.

Types of Sets

The sets are further categorised into different types, based on elements or types of elements. These different types of sets in basic set theory are:

- Finite set: The number of elements is finite
- Infinite set: The number of elements are infinite

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- Empty set: It has no elements
- Singleton set: It has one only element
- Equal set: Two sets are equal if they have same elements
- Equivalent set: Two sets are equivalent if they have same number of elements
- Power set: A set of every possible subset.
- Universal set: Any set that contains all the sets under consideration.
- Subset: When all the elements of set A belong to set B, then A is subset of B

Set Theory Formulas

- $n(A \cup B) = n(A) + n(B) - n (A \cap B)$
- $n(A\cup B)=n(A)+n(B)$ {when A and B are disjoint sets}
- $n(U)=n(A)+n(B)-n(A\cap B)+n((A\cup B)c)$
- $n(A\cup B)=n(A-B)+n(B-A)+n(A\cap B)$
- $n(A-B)=n(A\cap B)-n(B)$
- $n(A-B)=n(A)-n(A\cap B)$
- $n(Ac)=n(U)-n(A)$
- $n(PUQUR)=n(P)+n(Q)+n(R)-n(P\cap Q)-n(Q\cap R)-n(R\cap P)+n(P\cap Q\cap R)$

SET OPERATIONS

The four important set operations that are widely used are:

- Union of sets
- Intersection of sets
- Complement of sets
- Difference of sets

FUNDAMENTAL PROPERTIES OF SET OPERATIONS:

Like addition and multiplication operation in algebra, the operations such as union and intersection in set theory obeys the properties of associativity and commutativity. Also, the intersection of sets distributes over the union of sets.

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Sets are used to describe one of the most important concepts in mathematics i.e. functions. Everything that you observe around you, is achieved with mathematical models which are formulated, interpreted and solved by functions.

Problems and Solutions

Q.1: If U = {a, b, c, d, e, f}, A = {a, b, c}, B = {c, d, e, f}, C = {c, d, e}, find (A ∩ B) ∪ (A ∩ C).

Solution: A ∩ B = {a, b, c} ∩ {c, d, e, f}

A ∩ B = { c }

A ∩ C = { a, b, c } ∩ { c, d, e }

A ∩ C = { c }

∴ (A ∩ B) ∪ (A ∩ C) = { c }

Q.2: Give examples of finite sets.

Solution: The examples of finite sets are:

Set of months in a year

Set of days in a week

Set of natural numbers less than 20

Set of integers greater than -2 and less than 3

Q.3: If U = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, A = {3, 5, 7, 9, 11} and B = {7, 8, 9, 10, 11}, Then find (A – B)′.

Solution: A – B is a set of member which belong to A but do not belong to B

∴ A – B = {3, 5, 7, 9, 11} – {7, 8, 9, 10, 11}

A – B = {3, 5}

According to formula,

(A − B)′ = U – (A – B)

∴ (A − B)′ = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11} – {3, 5}

(A − B)′ = {2, 4, 6, 7, 8, 9, 10, 11}.

FORMALIZED SET THEORY

Elementary set theory can be studied informally and intuitively, and so can be taught in primary schools using Venn diagrams. The intuitive approach tacitly assumes that a set may be formed from the class of all objects satisfying any particular defining condition. This assumption gives rise to paradoxes, the simplest and best known of which are Russell's paradox and the Burali-Forti paradox. Axiomatic set theory was originally devised to rid set theory of such paradoxes.[note 1]

The most widely studied systems of axiomatic set theory imply that all sets form a cumulative hierarchy. Such systems come in two flavors, those whose ontology consists of:

Sets alone. This includes the most common axiomatic set theory, Zermelo–Fraenkel set theory with the axiom of choice (ZFC). Fragments of ZFC include:

Zermelo set theory, which replaces the axiom schema of replacement with that of separation;

General set theory, a small fragment of Zermelo set theory sufficient for the Peano axioms and finite sets;

Kripke–Platek set theory, which omits the axioms of infinity, powerset, and choice, and weakens the axiom schemata of separation and replacement.

Sets and proper classes. These include Von Neumann–Bernays–Gödel set theory, which has the same strength as ZFC for theorems about sets alone, and Morse–Kelley set theory and Tarski–Grothendieck set theory, both of which are stronger than ZFC.

The above systems can be modified to allow urelements, objects that can be members of sets but that are not themselves sets and do not have any members.

The New Foundations systems of NFU (allowing urelements) and NF (lacking them), associate with Willard Van Orman Quine, are not based on a cumulative hierarchy. NF and NFU include a "set of everything", relative to which every set has a complement. In these systems urelements matter, because NF, but not NFU, produces sets for which the axiom of choice does not hold. Despite NF's ontology not reflecting the traditional cumulative hierarchy and violating well-foundedness, Thomas Forster has argued that it does reflect an iterative conception of set.

Systems of constructive set theory, such as CST, CZF, and IZF, embed their set axioms in intuitionistic instead of classical logic. Yet other systems accept classical logic but feature a nonstandard membership relation. These include rough set theory and fuzzy set theory, in which the value of an atomic formula embodying the membership relation is not simply True or False. The Boolean-valued models of ZFC are a related subject.An enrichment of ZFC called internal set theory was proposed by Edward Nelson in 1977.

CONCLUSION

As set theory gained popularity as a foundation for modern mathematics, there has been support for the idea of introducing the basics of naive set theory early in mathematics education.
In the US in the 1960s, the New Math experiment aimed to teach basic set theory, among other abstract concepts, to primary school students, but was met with much criticism. The math syllabus in European schools followed this trend, and currently includes the subject at different levels in all grades. Venn diagrams are widely employed to explain basic set-theoretic relationships to primary school students (even though John Venn originally devised them as part of a procedure to assess the validity of inferences in term logic).
Set theory is used to introduce students to logical operators (NOT, AND, OR), and semantic or rule description (technically intensional definition[30]) of sets (e.g. "months starting with the

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letter A"), which may be useful when learning computer programming, since Boolean logic is used in various programming languages. Likewise, sets and other collection-like objects, such as multisets and lists, are common datatypes in computer science and programming.

In addition to that, sets are commonly referred to in mathematical teaching when talking about different types of numbers (the sets \mathbb {N} of natural numbers, \mathbb {Z} of integers, \mathbb {R} of real numbers, etc.), and when defining a mathematical function as a relation from one set (the domain) to another set (the range).

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