

CHAPTER 10

Set Theory and its Application in Mathematics

Dr. T. Chandrapusam

Nehru Arts and Science College, India

Mrs. S. Yamunadevi

Vivekanandha College of Arts and Sciences for Women, India

Mrs. T. Menahadevi

Vivekanandha College of Arts and Sciences for Women, India

Mrs. R. Rathna

Jai Shriram Engineering College, India

ABSTRACT

Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Mathematical logic is a subfield of mathematics exploring the applications of formal logic to mathematics. It bears close connections to meet mathematics, the foundations of mathematics, and theoretical computer science. The unifying themes in mathematical logic include the study of the expressive power of formal systems and the deductive power of formal proof systems. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics. The language of set theory can be used in the definitions of nearly all mathematical objects. The study investigates various types of set theory and their applications in solving real time problems. The main intent of the study is to explore different types of set theory and their dynamic usage in the different areas of problems. So the study highlights the dynamic ways in which different set theory are used. To do so, an exploratory research approach has been applied.

Keywords: *Set theory, Universal Set, Null Set, Finite Set, Infinite Set, Equivalent Set, Venn diagram and De Morgan's law.*

INTRODUCTION

Set theory is a mathematical abstract concerned with the grouping of sets of numbers that have commonality. For example, all even numbers make up a set, and all odd numbers comprise a set. All numbers that end in zero make up a set of numbers that can be divided by 10. Using and comparing sets enables the creation of theories and rules that have practically unlimited scope, whether in mathematics or applied to areas such as business. Applied to business operations, set theory can assist in planning and operations. Every element of business can be grouped into at least one set such as accounting, management, operations, production and sales. Within those sets are other sets. In operations, for example, there are sets of warehouse operations, sales operations and administrative

operations. In some cases, sets intersect -- as sales operations can intersect the operations set and the sales set.

Objective of the study:

The main objective of the study is to explore different types of set theory and their dynamic usage in the different areas of problems.

Methodology of the study:

Exploratory research approach has been used to conduct their search program successfully.

Definition of set 1.1:

A set is a collection of well-defined objects or elements having certain properties. Each object comprising a set is called element of the set. Usually, a set is denoted by the capital letters

A, B, C, D, etc and elements are denoted by the small letters a, b, c, d, etc and also the elements of the set are placed within a braces or second bracket.

Properties of set 1.2:

It should be well defined

It is the collection of all objects

Elements of set must be well-distinguished and independent.

Elements of set must be homogeneous.

Elements of set must be written second bracket { } and separated by commas (,)

Elements of a Set 1.3:

The objects used to form a set are called its element or its members. Generally, the elements of a set are written inside a pair of curly (idle) braces and are represented by commas. The name of the set is always written in capital letter.

Examples to find the elements or members of a set:

$$1. A = \{v, w, x, y, z\}$$

Here 'A' is the name of the set whose elements (members) are v, w, x, y, z.

2. If a set $A = \{3, 6, 9, 10, 13, 18\}$. State whether the following statements are 'true' or 'false':

$$i) 7 \in A \quad ii) 12 \notin A \quad iii) 13 \in A \quad iv) 9, 12 \in A \quad v) 12, 14, 15 \in A$$

Different Notations in Sets 1.4 :

\in

Belongs to

 \notin

Does not belongs

 \emptyset

A Null set or empty set

 \cup

Union of two sets

 \cap

Intersection of two sets

I or Set of integers $\{\dots\dots\dots, -2, -1, 0, 1, 2, \dots\dots\dots\}$ \mathbb{Z}

Set of all positive

integers Set

 \mathbb{Z}^+

all rational numbers

 \mathbb{W} Set of whole numbers = $\{0, 1, 2, 3, \dots\dots\dots\}$ \mathbb{R}^+ Set of all complex numbers \mathbb{C}

These are the different notations in

Different Types of Set 2.1:

Sets may be of various types. We give below a few of them...

Finite Set 2.1.2:

When the elements of a set can be counted by finite number of elements then the set is called a finite set. The following are the examples of finite sets:

 $A = \{ \}$ $B = \{ \}$ $C = \{ \}$

In all above sets the elements can be counted by a finite number. It should be denoted that a set containing a very large number of elements is also a finite set. Thus, the set of all human beings in Bangladesh, the set of all integers between -1 crore and +1 crore all finite sets.

Infinite Set 2.1.3:

If the elements of a set cannot be counted in a finite number, the set is called an infinite set. The following are examples of infinite sets:

$$A = \{1, 2, 3, 4, 5, \dots\}$$

$$B = \{x: x \text{ is an odd positive integer}\}$$

$$C = \{x: x \text{ is an even positive integer}\}$$

Empty or Null Set 2.1.4:

A Set containing only one element is called a singleton or a unit set. For example.. $A = \{x: x \text{ is neither prime nor composite}\}$

It is a singleton set containing one element, i.e., 1.

$$B = \{x: x \text{ is a whole number, } x < 1\}$$

this set contains only one element 0 and is a singleton set.

Equal Set 2.1.6:

Two sets A and B are said to be equal if every element of A is also an element of B, and every element of B is also an element of A ,

i.e. We find that $A=B=C$, as each set contains the same elements namely a, c, h, m, r irrespective of their order. Hence the sets are equal. It may be noted that the order of elements or the repetition of elements does not matter in set theory.

Let $A=\{2,3\}$, $B=\{3,2,2,3\}$ and $C=\{X:X^2-5X+6=0\}$. This $A=B=C$ since each element which belongs to anyone of the sets also belongs to the other two sets. Hence the sets are equal.

Equivalent Sets 2.1.7:

Here A and C are equal sets while A and B are equivalent sets. Let $A=\{a,b,c,d\}$ and $B=\{3,2,2,3\}$

Here the elements of A can be put into one to one correspondence with those of B .Thus

a,	b,	c,	d
3,	2,	2,	3

Hence $A \cong B$.

Subset 3.1:

If A and B are two sets, and every element of set A is also an element of set B, then A is

called a subset of B and we write it as $A \subseteq B$ or $B \supseteq A$

The symbol \subset stands for 'is a subset of' or 'is contained in'

Every set is a subset to itself, i.e., $A \subset A, B \subset B$.

Empty set is a subset of every set.

Symbol ' \subseteq ' is used to denote 'is a subset of' or 'is contained in'.

$A \subseteq B$ means A is a subset of B or A is contained in B.

$B \subseteq A$ means B contains A.

For example;

1. Let $A = \{2, 4, 6\}$ & $B = \{6, 4, 8, 2\}$

Here A is a subset of B

Since, all the elements of set A are contained in set B. But B is not the subset of A

Since, all the elements of set B are not contained in set A.

2. The set N of natural numbers is a subset of the set Z of integers and we write $N \subset Z$.

3. Let $A = \{2, 4, 6\}$

$B = \{x : x \text{ is an even natural number less than } 8\}$

Here $A \subset B$ and $B \subset A$. Hence, we can say $A = B$

4. Let $A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6, 7\}$

Here $A \not\subset B$ and also $B \not\subset A$ [$\not\subset$ denotes 'not a subset of']

Super Set 3.2:

Whenever a set A is a subset of set B, we say the B is a superset of A and we write,

$B \supseteq A$.

Symbol \supseteq is used to denote 'is a super set of' For example; $A = \{a, e, i, o, u\}$

$B = \{a, b, c, \dots, z\}$

Here $A \subseteq B$ i.e., A is a subset of B but $B \supseteq A$ i.e., B is a super set of A

Proper Subset 3.3:

If A and B are two sets, then A is called the proper subset of B if $A \subseteq B$ but $B \not\supseteq A$ i.e., $A \neq B$. The symbol ' \subset ' is used to denote proper subset. Symbolically, we write $A \subset B$. For

example;

1. $A = \{1, 2, 3, 4\}$

Here $n(A) = 4$ $B = \{1, 2, 3, 4, 5\}$

Here $n(B) = 5$

We observe that, all the elements of A are present in B but the element '5' of B is not present in A.

So, we say that A is a proper subset of B. Symbolically, we write it as $A \subset B$

2. $A = \{p, q, r\}$

$B = \{p, q, r, s, t\}$

Here A is a proper subset of B as all the elements of set A are in set B and also $A \neq B$.

Power Set 3.4:

The collection of all subsets of set A is called the power set of A. It is denoted by $P(A)$. In $P(A)$, every element is a set. For example;

If $A = \{p, q\}$ then all the subsets of A will be

$P(A) = \{\emptyset, \{p\}, \{q\}, \{p, q\}\}$

Number of elements of $P(A) = n[P(A)] = 4 = 2^2$

In general, $n[P(A)] = 2^m$ where m is the number of elements in set A.

Universal Set 3.5 :

A set which contains all the elements of other given sets is called a universal set. The symbol for denoting a universal set is U or ξ . For example;

1. If $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$ $C = \{3, 5, 7\}$

then $U = \{1, 2, 3, 4, 5, 7\}$

[Here $A \subseteq U$, $B \subseteq U$, $C \subseteq U$ and $U \supseteq A$, $U \supseteq B$, $U \supseteq C$]

2. If P is a set of all whole numbers and Q is a set of all negative numbers then the universal set is a set of all integers.

3. If $A = \{a, b, c\}$ $B = \{d, e\}$ $C = \{f, g, h, i\}$

then $U = \{a, b, c, d, e, f, g, h, i\}$ can be taken as universal set.

Disjoint Sets 3.6:

Two sets A and B are said to be disjoint, if they do not have any element in common. For example;

$A = \{x : x \text{ is a prime number}\}$

$B = \{x : x \text{ is a composite number}\}.$

Clearly, A and B do not have any element in common and are disjoint sets.

Family of sets 3.7:

If all the elements of a set of set themselves then such a set is called family of set. Example: If $P = \{X, Y\}$ then the set $\{\emptyset, \{x\}, \{b\}, \{x, y\}\}$ is the family of sets whose elements are subsets of the set P.

Operations on sets 4.1 :

The method of assigning such a new set is called operations on set. There are several fundamental operations for constructing new sets from given sets. The operations on sets are defined to develop algebra of sets. Types of set operations are given below :

Union of sets 4.1.2: The union of A and B, denoted $A \cup B$

Two sets can be "added" together. The union of A and B, denoted by $A \cup B$, is the set of all things that are members of either A or B.

Examples:

$$\{1, 2\} \cup \{1, 2\} = \{1, 2\}.$$

$$\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}.$$

$$\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$$

Some basic properties of unions:

$$A \cup B = B \cup A.$$

$$A \cup (B \cup C) = (A \cup B) \cup C.$$

$$A \subseteq (A \cup B). \quad \text{The union of A and B, denoted } A \cup B$$

$$A \cup A = A$$

$$A \cup \emptyset = A.$$

$$A \subseteq B \text{ if and only if } A \cup B = B.$$

Intersections 4.1.3:

A new set can also be constructed by determining which members two sets have "in common". The intersection of A and B, denoted by $A \cap B$, is the set of all things that are members of both A and B. If $A \cap B = \emptyset$, then A and B are said to be disjoint.

Examples:

$$\{1, 2\} \cap \{1, 2\} = \{1, 2\}.$$

$$\{1, 2\} \cap \{2, 3\} = \{2\}.$$

Some basic properties of intersections:

$$A \cap B = B \cap A.$$

$$A \cap (B \cap C) = (A \cap B) \cap C.$$

$$A \cap B \subseteq A.$$

$$A \cap A = A.$$

$$A \cap \emptyset = \emptyset. \quad \text{The intersection of A and B, denoted } A \cap B$$

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

Complements 4.1.4:

Two sets can also be "subtracted". The relative complement of B in A (also called the set-theoretic difference of A and B), denoted by $A \setminus B$ (or $A - B$), is the set of all elements that are members of A but not members of B. Note that it is valid to "subtract" members of a set that are not in the set, such as removing the element green from the set $\{1, 2, 3\}$; doing so has no effect.

In certain settings all sets under discussion are considered to be subsets of a given universal set U. In such cases, $U \setminus A$ is called the absolute complement or simply complements of A, and is denoted by A^c .

Examples:

$$\{1, 2\} \setminus \{1, 2\} = \emptyset.$$

$$\{1, 2, 3, 4\} \setminus \{1, 3\} = \{2, 4\}.$$

If U is the set of integers, E is the set of even integers, and O is the set of odd integers, then $U \setminus E = E^c = O$.

Some basic properties of complements:

$$A \setminus B \neq B \setminus A \text{ for } A \neq B.$$

$$A \cup A^c = U.$$

$$A \cap A^c = \emptyset.$$

$$(A^c)^c = A.$$

$$A \setminus A = \emptyset.$$

$$U^c = \emptyset \text{ and } \emptyset^c = U.$$

The relative complement of B in A

Relative complement of A in B

$B - A$ or $A \setminus B$

An extension of the complement is the symmetric difference, defined for sets A, B as

For example, the symmetric difference of

$\{7, 8, 9, 10\}$ and $\{9, 10, 11, 12\}$ is the set $\{7, 8, 11, 12\}$.

Difference of two sets 4.1.5:

If A and B are any two sets then the difference of A and B is the set of

elements which belong to A but do not belong to B. The difference of A and B is denoted by $A - B$ or $A \setminus B$. Symbolically we can write $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Cartesian product 5.1 :

A new set can be constructed by associating every element of one set with every element of another set. The Cartesian product of two sets A and B, denoted by $A \times B$ is the set of all ordered pairs (a, b) such that a is a member of A and b is a member of B.

Examples:

$$\{1, 2\} \times \{\text{red, white}\} = \{(1, \text{red}), (1, \text{white}), (2, \text{red}), (2, \text{white})\}.$$

$$\{1, 2\} \times \{\text{red, white, green}\} = \{(1, \text{red}), (1, \text{white}), (1, \text{green}), (2, \text{red}), (2, \text{white}), (2, \text{green})\}.$$

$$\{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}.$$

$$\{a, b, c\} \times \{d, e, f\} = \{(a, d), (a, e), (a, f), (b, d), (b, e), (b, f), (c, d), (c, e), (c, f)\}.$$

Some basic properties of Cartesian products:

$$A \times \emptyset = \emptyset.$$

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

$$(A \cup B) \times C = (A \times C) \cup (B \times C).$$

Let A and B be finite sets; then the cardinality of the Cartesian product is the product of the cardinalities:

$$|A \times B| = |B \times A| = |A| \times |B|.$$

VENN DIAGRAM 6.1 :

A Venn diagram is a diagram that shows all possible logical relations between finite collections of different sets. These diagrams depict elements as points in the plane, and sets as regions inside closed curves. A Venn diagram consists of multiple overlapping closed curves, usually circles, each representing a set. The points inside a curve labeled S represent elements of the set S , while points outside the boundary represent elements not in the set S . Thus, for example, the set of all elements that are members of both sets S and T , $S \cap T$, is represented visually by the area of overlap of the regions S and T . In Venn diagrams the curves are overlapped in every possible way, showing all possible relations between the sets. They are thus a special case of Euler diagrams, which do not necessarily show all relations. Venn diagrams were conceived around 1880 by John Venn. They are used to teach elementary set theory, as well as illustrate simple set relationships in probability, logic, statistics, linguistics and computerscience.

A Venn diagram in which in addition the area of each shape is proportional to the number of elements it contains is called an area-proportional or scaled Venndiagram.

Intersection of two sets

union of two sets

Relative complement of A in B

$A \text{ Bor } A \text{ Bc}$

De Morgan's law 6.1.2:

a) DeMorgan's lawson complementofsets

The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements. These are called De Morgan's laws.

For any two finite sets A and B ;

$(A \cup B)^c = A^c \cap B^c$ (which is a De Morgan's law of union).

$(A \cap B)^c = A^c \cup B^c$ (which is a De Morgan's law of intersection).

(i) Proof that: $(A \cup B)^c = A^c \cap B^c$

Let $x \in (A \cup B)^c$, here x be an arbitrary element of $(A \cup B)^c$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

Therefore, $(A \cup B)^c \subseteq A^c \cap B^c$ (1)

Again, let $y \in A^c \cap B^c$, here y be an arbitrary element of $A^c \cap B^c$

$\Rightarrow y \in A^c$ and $y \in B^c$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)^c$

Therefore, $A^c \cap B^c \subseteq (A \cup B)^c$ (2)

Now combine (1) and (2) we get $(A \cup B)^c = A^c \cap B^c$ Proved.

(ii) Proof that $(A \cap B)^c = A^c \cup B^c$

Let $x \in (A \cap B)^c$, here x be an arbitrary element of $(A \cap B)^c$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

Therefore, $(A \cap B)^c \subseteq A^c \cup B^c$ (1)

Again, let $y \in A^c \cup B^c$ here y be an arbitrary element of $A^c \cup B^c$

$\Rightarrow y \in A^c$ or $y \in B^c$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)^c$

Therefore, $A^c \cup B^c \subseteq (A \cap B)^c$ (2)

Now combine (1) and (2) we get $(A \cap B)^c = A^c \cup B^c$ Proved

b) DeMorgan's laws on difference of sets

Statement: Let A , B and C be any two sets then

$$(i) \quad A - (B \cup C) = (A - B) \cap (A - C)$$

$$(ii) \quad A - (B \cap C) = (A - B) \cup (A - C)$$

DeMorgan's first laws on difference of sets

Statement: Difference of a union is the intersection of difference i.e.

$$A - (B \cup C) = (A - B) \cap (A - C)$$

Proof: Let $x \in A - (B \cup C)$, here x be an arbitrary element of $A - (B \cup C)$.

$$\Rightarrow x \in A \text{ and } x \notin (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (x \notin B \text{ and } x \notin C)$$

$$\Rightarrow (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)$$

$$\Rightarrow x \in (A - B) \text{ and } x \in (A - C)$$

$$\Rightarrow x \in (A - B) \cap (A - C)$$

$$\text{Therefore } A - (B \cup C) \subseteq (A - B) \cap (A - C) \quad (1)$$

Again let $y \in (A - B) \cap (A - C)$ here y be an arbitrary element of $(A - B) \cap (A - C)$

$$\Rightarrow y \in (A - B) \text{ and } y \in (A - C)$$

$$\Rightarrow (y \in A \text{ and } y \notin B) \text{ and } (y \in A \text{ and } y \notin C)$$

$$\Rightarrow y \in A \text{ and } (y \notin B \text{ and } y \notin C)$$

$$\Rightarrow y \in A \text{ and } y \notin (B \cup C)$$

$$\Rightarrow y \in A - (B \cup C)$$

$$\text{Therefore } (A - B) \cap (A - C) \subseteq A - (B \cup C) \quad (2)$$

Now combine (i) and (ii) we get,

$$A - (B \cup C) = (A - B) \cap (A - C) \quad \text{Proved}$$

DeMorgan's first law on difference of sets

Statement: Difference of a intersection is the union of difference i.e.

$$A - (B \cap C) = (A - B) \cup (A - C)$$

Proof : Let $x \in A - (B \cap C)$, here x be an arbitrary element of $A - (B \cap C)$

$$\Rightarrow x \in A \text{ and } x \notin (B \cap C)$$

$$\Rightarrow x \in A \text{ and } (x \notin B \text{ or } x \notin C)$$

$$\Rightarrow (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)$$

$$\Rightarrow x \in (A - B) \text{ or } x \in (A - C)$$

$$\Rightarrow x \in (A - B) \cup (A - C)$$

Therefore $A - (B \cap C) \subseteq (A - B) \cup (A - C)$ (i)

Again let $y \in (A - B) \cup (A - C)$ here y be an arbitrary element of $(A - B) \cup (A - C)$

$\Rightarrow y \in (A - B)$ or $y \in (A - C)$

$\Rightarrow (y \in A \text{ and } y \notin B)$ or $(y \in A \text{ and } y \notin C)$

$\Rightarrow y \in A \text{ and } (y \notin B \text{ or } y \notin C)$

$\Rightarrow y \in A \text{ and } y \notin (B \cap C)$

$\Rightarrow y \in A - (B \cap C)$

Therefore $(A - B) \cup (A - C) \subseteq A - (B \cap C)$ (ii)

Now combine (i) and (ii) we get,

$A - (B \cap C) = (A - B) \cup (A - C)$ Proved

SOME PROVED ON SET OPERATIONS 6.1.3 :

Prove-1: Let A , B , and C be any three sets, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: Let $x \in A \cup (B \cap C)$, here x be an arbitrary element of $A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $\{x \in B \text{ and } x \in C\}$

$\Rightarrow \{x \in A \text{ or } x \in B\}$ and $\{x \in A \text{ or } x \in C\}$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ (1)

Again

Let $y \in (A \cup B) \cap (A \cup C)$, here y be an arbitrary element of $(A \cup B) \cap (A \cup C)$.

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow \{y \in A \text{ or } y \in B\}$ and $\{y \in A \text{ or } y \in C\}$

$\Rightarrow y \in A$ or $\{y \in B \text{ and } y \in C\}$

$\Rightarrow y \in A \cup \{y \in (B \cap C)\}$

$\Rightarrow y \in A \cup (B \cap C)$

Therefore, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ (2)

Now combine (1) and (2) we get,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad \text{Proved}$$

Prove-2:

Let A, B, and C be any three sets, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Proof: let $x \in A \cap (B \cup C)$, here x be an arbitrary element of $A \cap (B \cup C)$

$$\Rightarrow x \in A \text{ and } x \in (B \cup C)$$

$$\Rightarrow x \in A \text{ and } \{x \in B \text{ or } x \in C\}$$

$$\Rightarrow \{x \in A \text{ and } x \in B\} \text{ or } \{x \in A \text{ and } x \in C\}$$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

Therefore, $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ (1)

Again

Let $y \in (A \cap B) \cup (A \cap C)$, here y be an arbitrary element of $(A \cap B) \cup (A \cap C)$

$$\Rightarrow y \in (A \cap B) \text{ or } y \in (A \cap C)$$

$$\Rightarrow \{y \in A \text{ and } y \in B\} \text{ or } \{y \in A \text{ and } y \in C\}$$

$$\Rightarrow y \in A \text{ and } \{y \in B \text{ or } y \in C\}$$

$$\Rightarrow y \in A \text{ and } \{y \in (B \cup C)\}$$

$$\Rightarrow y \in A \cap (B \cup C)$$

Therefore $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ (2)

Now combine (1) and (2) we get,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \text{Proved}$$

Prove-3: If A and B are two sets, then prove that

$$A - B \subseteq A \text{ (ii) } A \cap (B - A) = \emptyset$$

Proof: (i) let $x \in A - B$, here x be an arbitrary element of $A - B$

$$\Rightarrow x \in A \text{ and } x \notin B$$

$$\Rightarrow x \in A \text{ or } x \notin B$$

$\Rightarrow x \in A$ [x is an element of set A , not set B]

$\Rightarrow A - B \subseteq A$ Proved

Proof: (ii) let $x \in A \cap (B - A)$, here x be an arbitrary element of $A \cap (B - A)$

$\Rightarrow x \in A$ and $x \in (B - A)$

$\Rightarrow x \in A$ and ($x \in B$ and $x \notin A$)

$\Rightarrow (x \in A \text{ and } x \notin A) \text{ and } x \in B$

$\Rightarrow \phi$ and $x \in B$

$\Rightarrow \phi$

Therefore $A \cap (B - A) \subseteq \phi$ (1)

Since ϕ is a subset of every set Therefore $\phi \subseteq A \cap (B - A) \dots (2)$

Now combine (1) and (2) we get,

$A \cap (B - A) = \emptyset$ Proved

CONCLUSION:

The theory of sets has been the base for the foundation of mathematics and so is considered as one of the most significant branches in mathematics. The fact that any mathematical concept can be interpreted with the help of set theory has not only increased its versatility but has established this theory to be the universal language of mathematics. In the recent past, a relook to the concept of uncertainty in science and mathematics has brought in paradigmatic changes. Set theory provides the basis of topology, the study of sets together with the properties of various collections of subsets.

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